Non-Gaussian Signatures of High Energy Physics in the CMB

Amjad Ashoorioon

Uppsala University

May 14th, 2011

A. Ashoorioon & Gary Shiu, JCAP03(2011)025, arXiv:1012.3392 [astro-ph.CO]

A. Ashoorioon, D. Chialva & U. Danielsson, arXiv:1104.2338 [hep-th]

Calm Excited States

The EOM for perturbations in de-Sitter space:

$$u_k'' + (k^2 - \frac{2}{\eta^2})u_k = 0$$

whose solution is

$$u_k(\eta) = \alpha_k\ (-\eta)^{1/2} H^{(1)}_{3/2}(-k\eta) + \beta_k\ (-\eta)^{1/2} H^{(2)}_{3/2}(-k\eta)$$

The Wronskian condition gives

$$|\alpha_k|^2 - |\beta_k|^2 = \frac{\pi}{4}$$

The power spectrum for the general solution

$$P_S = \frac{|\alpha_k - \beta_k|^2 H^2}{\pi^3 \epsilon} \quad \Longrightarrow \quad |\alpha_k - \beta_k|^2 = \frac{\pi}{4}$$

$$\begin{aligned} \alpha_k &= x_1 + i x_2 \\ \beta_k &= y_1 + i y_2 \end{aligned}$$

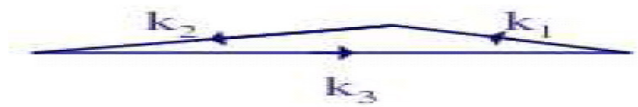
$$\begin{cases} x_1^{(1)} = y_1 - y_2\ \mathrm{sign}(y_1)\dfrac{\sqrt{\pi}}{2\sqrt{y_1^2 + y_2^2}} \\ x_2^{(1)} = y_2 + \dfrac{|y_1|\sqrt{\pi}}{2\sqrt{y_1^2 + y_2^2}} \end{cases} \qquad \begin{cases} x_1^{(2)} = y_1 + y_2\ \mathrm{sign}(y_1)\dfrac{\sqrt{\pi}}{2\sqrt{y_1^2 + y_2^2}} \\ x_2^{(2)} = y_2 - \dfrac{|y_1|\sqrt{\pi}}{2\sqrt{y_1^2 + y_2^2}} \end{cases}$$

$$\sigma \equiv |\beta_0| \le \min\left\{ \frac{\sqrt{\varepsilon} H M_P}{\Lambda^2}, \frac{\sqrt{\varepsilon \eta'} H M_P}{\Lambda^2} \right\}$$

Holman & Tolley (2007)

σ could be *O(1)* in some inflationary models.

- When the initial state is excited, the contribution of flattened configurations are enhanced



Flattened/Folded

$$\left.\frac{\Delta\langle \zeta_{\vec{k_1}} \zeta_{\vec{k_2}} \zeta_{\vec{k_3}} \rangle}{\langle \zeta_{\vec{k_1}} \zeta_{\vec{k_2}} \zeta_{\vec{k_3}} \rangle}\right|_{\tilde{k}_j = 0} \approx i C_4 k_t \eta_0 = i C_4 \frac{k_t}{a(\eta_0) H}$$

The result is enhanced by the presence of the ratio of physical momentum to the Hubble scale at the beginning of inflation.

$$\begin{aligned} C_4 &= \frac{i\pi^2}{8} \Im(\alpha_k \overline{\beta_k}) \\ &= \frac{i\pi^2}{8} (x_2 y_1 - x_1 y_2) \end{aligned}$$

$$\begin{aligned} C_4 &= \pm \frac{i\pi^2}{16} \left(\frac{y_1 |y_1| \sqrt{\pi}}{\sqrt{y_1^2 + y_2^2}} + \frac{y_2^2 \ \mathrm{sign}(y_1) \sqrt{\pi}}{\sqrt{y_1^2 + y_2^2}} \right) \\ &= \pm \frac{i\pi^{\frac{5}{2}}}{16} \sqrt{y_1^2 + y_2^2} \ \mathrm{sign}(y_1) \end{aligned}$$

Excited States with No Enhancement for the Flattened Configurations

$$|\alpha_k|^2 - |\beta_k|^2 = \frac{\pi}{4}$$

$$C_4 = (|\alpha_k|^2 - |\beta_k|^2)(|\alpha_k - \beta_k|^2)(\alpha_k\overline{\beta_k} - \overline{\alpha_k}\beta_k) = 0$$

One can identify a two-parameter of family states parameterized in the following way:

$$x_2 = y_2 \left(\frac{\frac{\pi}{4} + y_1^2 + y_2^2}{y_1^2 + y_2^2} \right)^{1/2}$$

$$x_1 = y_1 \left(\frac{\frac{\pi}{4} + y_1^2 + y_2^2}{y_1^2 + y_2^2} \right)^{1/2}$$

$$P_S = \frac{H^2}{(2\pi)^2\epsilon} \left(\frac{\pi - 4\sqrt{\pi + 4|\beta_k|^2}|\beta_k| + 8|\beta_k|^2}{\pi} \right) \xrightarrow[|\beta_k| << 1]{} P_S \simeq \frac{H^2}{4\pi^2\epsilon}(1 - \frac{4}{\sqrt{\pi}}|\beta_k|)$$

Calm Excited States in DBI Inflation

- The same excited states that leave the power spectrum invariant for slow-roll inflation, do the same for DBI inflation/

- Magnitude of enhancement for flattened configuration for such calm excited states:

$$\Delta\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle|_{\tilde{k}_j\to 0} = \mathcal{E}_2 f_3(k_i)(k_t\eta_0)^3 + \mathcal{E}_3 f_2(k_i)(k_t\eta_0)^2 + \mathcal{E}_2 f_1(k_i)k_t\eta_0$$

$$\begin{aligned} \mathcal{E}_2 &= \mp\frac{\pi^{\frac{3}{2}}}{4}\mathrm{sign}(y_1)|\beta_k| \\ \mathcal{E}_3 &= -\frac{3}{2}\pi|\beta_k|^2. \end{aligned}$$

- This conclusion for the amount of non-Gaussianity is different from what one would obtain assuming $\alpha_k \simeq 1$

P. D. Meerberg, J. P. Van der Schaar, M. Jackson (2009)

One may wonder if there are excited states that do not leave any enhancement for flattened configurations i $\mathcal{E}_2 \propto C_4 = 0$ ion?

$$\Delta\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle|_{\tilde{k}_j\to 0} = \mathcal{E}_2 \cancelto{0}{f_3}(k_i)(k_t\eta_0)^3 + \mathcal{E}_3 f_2(k_i)(k_t\eta_0)^2 + \mathcal{E}_2 \cancelto{0}{f_1}(k_i)k_t\eta_0$$

$$\mathcal{E}_3 \simeq \frac{\pi^{3/2}|\beta_k|}{4} + \cdots$$

$$\left.\frac{\Delta\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle}{\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle}\right|_{\tilde{k}_j\to 0} \simeq |\beta_k|k_t\eta_0$$

Now if one assumes that $\beta_k \approx \frac{\sqrt{\varepsilon\eta'}HM_P}{\Lambda^2}$ and $k_t\eta_0 \approx \frac{\Lambda}{H}$

$$\left.\frac{\Delta\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle}{\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle}\right|_{\tilde{k}_j\to 0} \approx \sqrt{\varepsilon\eta'}\,\frac{M_P}{\Lambda}$$

with $\epsilon \simeq \eta' \sim 10^{-2}$, the enhancement is “observationally” absent if

$$\Lambda \geq 10^{-2} M_P$$

Modified Dispersion Relation

$$u''_{\vec{k}} + \left(\omega(\eta,\vec{k})^2 - \frac{z''}{z}\right)u_{\vec{k}} = 0$$

$u_{\vec{k}}(\eta)$: Mukhanov-Sasaki variable

$$u_{\vec{k}}(\eta) = z\,\zeta_{\vec{k}}(\eta) \qquad z = \frac{a\dot{\phi}}{H}$$

- In the standard QFT formalism, $\omega(\eta,\vec{k}) = k \equiv \left|\vec{k}\right|$

- The dispersion relations were motivated by some condensed matter expectation from QG, that were used to study the trans-Planckian issue in Black hole physics.

$$\omega^2 = F^2(p)$$

Unruh (1996)
Jacobson, Corley (1996,1997)
Horava (2009)

- To implement this in an expanding background:

$$k^2 \Rightarrow a^2(\eta)F^2\left(\frac{k}{a(\eta)}\right)$$

Brandenberger & Martin (2001)
Brandenberger (2002)

we will focus on Corley-Jacobson dispersion relation with positive quartic correction

$$k^2 \Rightarrow a^2(\eta)F^2\left(\frac{k}{a(\eta)}\right) = k^2 + b_1\frac{k^4}{p_c^2 a^2(\eta)}$$

EOM for the perturbations in de-Sitter space

$$u_k''(\eta) + (k^2 + \epsilon^2 k^4 \eta^2 - \frac{2}{\eta^2})u_k = 0$$

$$\epsilon \equiv \frac{b_1^{1/2} H}{p_c}$$

Solution

$$\begin{aligned} u_k &= \frac{C_1}{\sqrt{-\eta}} \mathrm{WW}\left(\frac{i}{4\epsilon}, \frac{3}{4}, -i\epsilon k^2 \eta^2\right) + \frac{C_2}{\sqrt{-\eta}} \mathrm{WW}^*\left(\frac{i}{4\epsilon}, \frac{3}{4}, -i\epsilon k^2 \eta^2\right) \\ &= \frac{C_1}{\sqrt{-\eta}} \mathrm{WW}\left(\frac{i}{4\epsilon}, \frac{3}{4}, -i\epsilon k^2 \eta^2\right) + \frac{C_2}{\sqrt{-\eta}} \mathrm{WW}\left(\frac{-i}{4\epsilon}, \frac{3}{4}, i\epsilon k^2 \eta^2\right) \end{aligned}$$

Requiring the mode behaves like positive frequency WKB mode

$$\begin{aligned} u_k(\eta) &\simeq \frac{1}{\sqrt{2\omega(\eta)}} \exp(-i \int^{\eta} \omega(\eta) d\eta') \\ &= \frac{1}{k\sqrt{-2\epsilon\eta}} \exp(\frac{i\epsilon k^2 \eta^2}{2}) \end{aligned}$$

$\Longrightarrow$ $C_2 = 0$

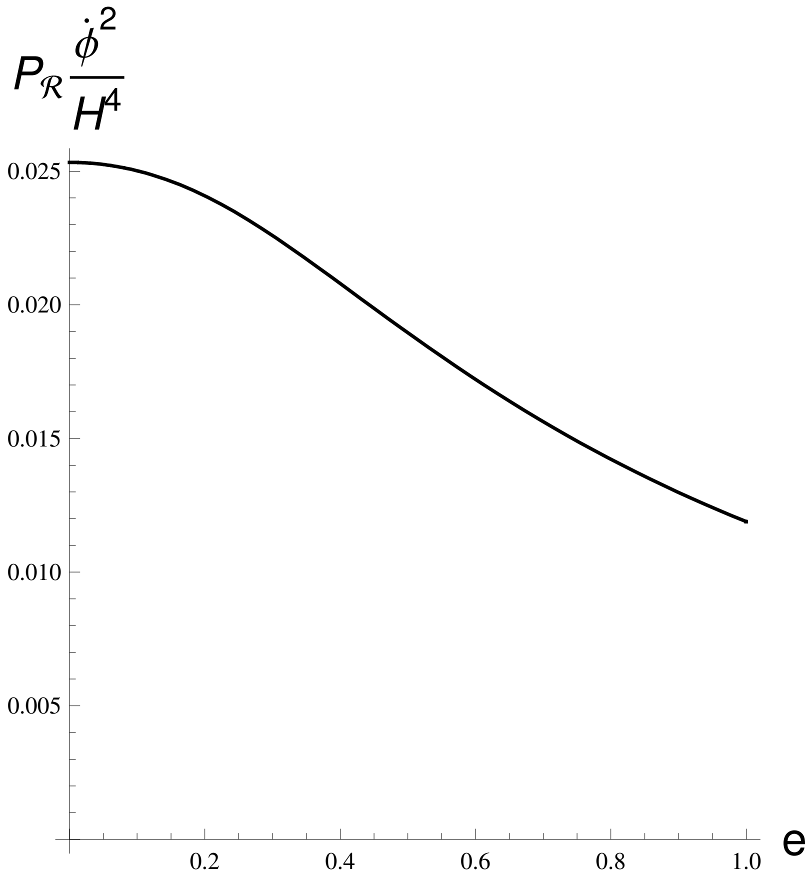
$$u(\eta)u'^*(\eta) - u^*(\eta)u'(\eta) = i$$

$\Longrightarrow$ $C_1 = \dfrac{\exp(\frac{-\pi}{8\epsilon})}{\sqrt{2\epsilon k}}$

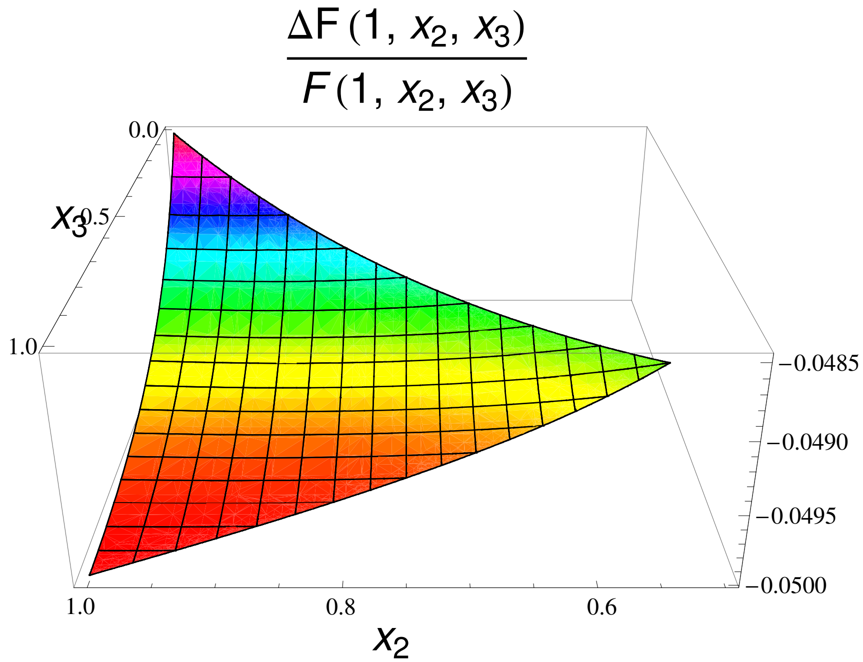
$$P_{\mathcal{R}}(\epsilon) = \frac{H^4}{\dot{\phi}^2} \frac{\exp(-\frac{\pi}{4\epsilon})}{16\pi\epsilon^{3/2}\Gamma(\frac{5}{4} - \frac{i}{4\epsilon})\Gamma(\frac{5}{4} + \frac{i}{4\epsilon})}$$

The leading order correction to the power spectrum is proportional to ϵ^2

$$P_{\mathcal{R}}(\epsilon) \simeq \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \left(1 - \frac{5}{4}\epsilon^2\right)$$



$$\frac{\Delta F(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3})}{F(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3})} = (-5 + \frac{k_c^3}{k_t^3} - \frac{k_s^2}{2k_t^2})\epsilon^2$$



The largest modification of the nonlinear dispersion relation occurs for the equilateral configurations.

Comparison with the gluing method:

$(k\|\eta\|)^2 < 2$	region I
$2 < (k\|\eta\|)^2 < \epsilon^{-2}$	region II
$(k\|\eta\|)^2 > \epsilon^{-2}$	region III

Martin & Brandenberger (2001)

- Knowing that the evolution in the region III leads to excited state as the initial condition for region II, one would expect to observe enhancement for the folded configurations!

- In fact,

$$u_k = \frac{\alpha_k}{\sqrt{2k}} e^{-ik\eta} + \frac{\beta_k}{\sqrt{2k}} e^{ik\eta}$$

$$\begin{cases} \alpha_k = e^{-\frac{i}{2\epsilon}} \left(1 + i\frac{\epsilon}{4}\right) \\ \beta_k = -ie^{i\frac{3}{2\epsilon}} \left(\frac{\epsilon}{4}\right) \end{cases}$$

$$\frac{\Delta F}{F} \approx \left| C_4 k \eta_c \right| \approx 1$$

$$\eta_c = -\frac{1}{\epsilon k}$$

- The gluing method predicts enhancement to be of order one for the bispectrum!

- Calculating the power spectrum using the gluing method also shows that $\frac{\Delta P_S}{P_S} \approx \epsilon$!

- This specific example shows the incapability of the gluing method in capturing the correct modification.

Thank you!